Using the Ratio Test

The ratio test for convergence is another way to tell whether a sum of the form $\sum_{n=n_0}^{\infty} a_n$, with $a_n > 0$ for all n, converges or diverges. To perform the ratio test we find the ratio $\frac{a_{n+1}}{a_n}$ and let:

$$L = \lim_{n\to\infty} \frac{a_{n+1}}{a_n}.$$

The test has three possible outcomes:

$L < 1$	$\Rightarrow$	The series converges.
$L > 1$	$\Rightarrow$	The series diverges.
$L = 1$		No conclusion; the series may converge or diverge.

Apply the ratio test to each of the following series. Note that not all series satisfy the conditions needed to apply this form of the ratio test.

a) $\displaystyle\sum_{n=0}^{\infty} \frac{1}{\sqrt[3]{n}}$

b) $\displaystyle\sum_{n=0}^{\infty} \frac{(n!)^2}{(2n)!}$

c) $\displaystyle\sum_{n=0}^{\infty} \frac{(-3)^n}{n!}$

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a) $\lim_{n\to\infty} \frac{1}{\sqrt[3]{n+1}} \cdot \frac{\sqrt[3]{n}}{1}$

$= 1$

$\therefore \sum_{n=0}^{\infty} \frac{1}{\sqrt[3]{n}}$, No conclusion.

b) $\lim_{n\to\infty} \frac{(n+1)!^2}{(2(n+1))!} \cdot \frac{(2n)!}{(n!)^2}$

$= \lim_{n\to\infty} \frac{(n+1)^2 n!^2}{(2n+2)!} \cdot \frac{(2n)!}{(n!)^2}$

$= \lim_{n\to\infty} \frac{(n+1)^2}{(2n+2)(2n+1)}$

$= \lim_{n\to\infty} \frac{n+1}{2(2n+1)}$

$= \frac{1}{2}\frac{n}{2n}$

$= \frac{1}{4} < 1$

$\therefore \sum_{n=0}^{\infty} \frac{(n!)^2}{(2n)!}$ converges.

c) $a_n = \frac{(-3)^n}{n!} < 0$ for $n=3$.

$\Rightarrow$ ratio test is not applicable

$|a_n| = \left|\frac{(-3)^n}{n!}\right| > 0$ for all n

$\lim_{n\to\infty} \left|\frac{(-3)^{n+1}}{(n+1)!} \cdot \frac{n!}{(-3)^n}\right|$

$= \lim_{n\to\infty} \left|\frac{(-3)}{n+1}\right|$

$= 0 < 1$

$\Rightarrow \sum_{n=0}^{\infty} \left|\frac{(-3)^n}{n!}\right|$ converges

$\sum_{n=0}^{\infty} \frac{(-3)^n}{n!} < \sum_{n=0}^{\infty} \left|\frac{(-3)^n}{n!}\right|$

$\therefore \sum_{n=0}^{\infty} \frac{(-3)^n}{n!}$ converges.